

# **FORCE LIMIT SPECIFICATIONS VS. DESIGN LIMIT LOADS IN VIBRATION TESTING**

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## **Abstract**

Flight equipment is exposed to random vibration excitations during launch and is functionally designed to survive a shaker random vibration test. In the test, the random vibration design levels will be applied at the equipment mounting interface and will be force limited to reduce over-testing at shaker hardmount resonance frequencies. As is commonly practiced for heavier equipment, the equipment also is designed to the structural flight limit load. The philosophy of the limit load factors (LLFs) or so-called the Mass Acceleration Curve (MAC) has been adopted over many years for use in the preliminary structural design of spacecraft and flight equipment. The purpose of the work presented herein is to discuss the results of force limit notching during vibration testing with respect to the traditional limit load design criteria. By using a single-degree-of-freedom (SDOF) system approach, this work shows that with an appropriate force specification the notched response due to force limiting will result in loads comparable with the structural design limit criteria. A simplified formula is presented to predict the test load limits, based only upon estimates for the first significant resonance of the equipment and the semi-empirical force specification factor,  $C^2$ . The work is currently being expanded to include two and multi-degree-of-freedom vibratory systems.

## **Introduction**

Flight equipment is exposed to random vibration excitations during launch and is functionally designed to survive shaker random vibration testing. In the testing, the random vibration design levels will be applied at the equipment mounting interface. For lightweight aerospace structures, the mechanical impedance of equipment and of the mounting structure are typically comparable, so that the vibration of the combined structure and load involves modest interface forces and responses. Most of the high amplification resonances and mechanical failures in conventional vibration tests are test artifacts associated with the essentially infinite mechanical impedance and unlimited force capability of the shaker. An improved vibration testing technique has been recently developed [Ref. 1,2] and applied to eliminate overtesting caused by the infinite mechanical impedance of the shaker in conventional vibration tests. With the newly developed technique, the acceleration input is automatically notched at the resonance frequencies of the test item by specifying a force limit in order to limit the test loads to that predicted for flight.

## Force Limit Specification in Random Vibration Testing

Implementation of force-limited vibration testing requires derivation of a force limit specification. The flight force at the equipment interface, which may be derived from Norton and Thevenin's equivalent electrical circuit theorems, can be written as follows:

$$S_{FF}(f) = \left| \frac{Z_{EE}(f) Z_{SS}(f)}{Z_{EE}(f) + Z_{SS}(f)} \right|^2 S_{AA}(f) \quad (1)$$

where

- $S_{FF}(f)$  = Interface Force Spectrum
- $Z_{SS}(f)$  = Input Impedance of Support Structures
- $Z_{EE}(f)$  = Input Impedance of Component or Equipment
- $S_{AA}(f)$  = Acceleration Input Spectrum

Input impedances in the above equation are specified in terms of the "force/acceleration" format. For a rigid body system, the impedance is equal to its mass only. For complex structural systems the force limit values must be calculated from measurements or analyses of the flight mounting structure and the test items mechanical impedances. An alternate approach to compute the driving force spectrum could be achieved by the replacement of the impedance term by the load dynamic mass.

$$S_{FF}(f) = |M_D(f)|^2 S_{AA}(f) \quad (2)$$

In this expression, the load dynamic mass,  $M_D(f)$ , is a frequency response function (FRF) that includes mass, damping, and stiffness effects. The frequency dependence is shown explicitly to emphasize the relationship between force and acceleration applied at each frequency. Since little flight vibratory force data are available and due to structural complexities of space vehicles, precise analytical approaches to obtain the parameters defined in the above equation are not practical. In order to validate Equation (2), several approximate methods [Ref. 3,4] along with measured data were used to predict the force spectra quantitatively. The semi-empirical method [Ref. 5] was also developed to simplify the development of shaker force limited vibration testing criteria. In this simplified method, the input force spectrum at the fundamental resonance frequency is properly enveloped by multiplying the total mass,  $M$ , of the test item by the input acceleration spectral density specification and by a constant,  $C$ . The required force spectrum value is then related to the shaker control acceleration spectrum as

$$\begin{aligned} S_{FF}(f) &= C^2 M^2 S_{AA}(f_0), & f < f_0 \\ S_{FF}(f) &= C^2 M^2 S_{AA}(f_0) / (f / f_0)^n, & f \geq f_0 \end{aligned} \quad (3)$$

where  $f_0$  is the fundamental resonance of the test item on the shaker. Some judgment and reference to previous test data for similar configurations must be considered to choose the constant value of  $C$  and the roll-off ratio,  $n$ , in the above equation.

Semi-empirical force specifications require only the acceleration specification and data from a low-level vibration pretest and are, therefore, much simpler to determine than previously described force limits based on analytical models and measurements of the mounting structure mechanical impedance. The force limit conservatism is dependent on the chosen constant or so-called fudge factor,  $C^2$ . In normal conditions, as high as  $C^2 = 5$  for directly mounted lightweight loads, and  $C^2 = 2$  for strut mounted heavier equipment may be considered. The simplified method to derive the test force specification has been successfully used in many JPL spacecraft and component tests [Ref. 5,6,7& 8].

## Quasi-Static Acceleration for Equipment Structural Design

The design limit loads for aerospace equipment are usually given in terms of the “quasi-static” acceleration of the center-of-gravity (C.G.) of the hardware. The preliminary Limit Load Factor (LLF) or so-called the Mass Acceleration Curve (MAC) has been adapted over many years for use in the preliminary structural design of spacecraft structures and equipment [Ref. 9]. The accelerations shown on the MAC, as illustrated in Figure 1 for a typical spacecraft, should be applied on the C.G. of the equipment in the low frequency range (i.e., usually up to 80 or 100 Hz). In practice, the MAC could bound the equipment launch vibration loads for all frequency ranges, except for very lightweight and high stiffness payload equipment.

During the equipment vibration qualification tests, loads induced in structural elements are normally not allowed to exceed the specified limit loads. In cases where the shaker test induced loads would otherwise exceed the limit loads of the structure, input notching or other response-limiting measures must be taken to protect the flight hardware being tested. The purpose of this paper is to study whether the force limiting has accomplished the notching requirement to limit the equipment structural response in low frequency vibration tests to something less than the design load.

## Acceleration Responses under Force-Limited Excitation

To illustrate the concept of the force limited vibration excitation, consider first a single-degree-of-freedom (SDOF) system as illustrated in Figure 2, where a single mass  $M$  is suspended from a moving support by means of a linear spring in parallel with a linear dashpot. The system is subjected to a constant wide-band random excitation of its base. The narrow band response of the SDOF due to random excitation is very similar to sinusoidal motion with randomly varying amplitude. The response acceleration spectral density  $ASD_x$  of the mass is given by

$$ASD_x = S_{AA}(f) |T(f)|^2 \quad (4)$$

where  $|T(f)|$  is the sinusoidal transmissibility for such a system. The area under the transmissibility curve is finite so that for a constant input acceleration spectrum, the root mean square (RMS) acceleration response of the mass can be given in closed form [Ref. 10].

$$\sigma_x = [0.5 \pi f_o Q S_{AA}(f_o)]^{1/2} \quad (5)$$

where  $Q$  is the dynamic amplification factor and  $f_0$  is the resonant frequency of the system. The procedure to determine the RMS acceleration for one single resonant mode can be applied to more practical cases, which usually exhibit many resonant peaks. As shown in Figure 3, the response acceleration spectral density curve, in the vicinity of resonance of a single resonant mode, can be replaced by an equivalent narrow band with an effective bandwidth equal to

$$\Delta f_{\text{eff}} = 0.5 \pi f_0 / Q \quad (6)$$

and a constant acceleration spectral density equal to the maximum value at resonance,  $ASD_{\text{max}}$ . The effective bandwidth is equal to the so-called half-power bandwidth  $f_0 / Q$  of the resonant system times a correction factor  $\pi/2$  which accounts for response outside this bandwidth. The mean square acceleration within this equivalent band is then equal the product of the above two quantities. Since  $ASD_{\text{max}} = S_{AA}(f_0) Q^2$ , the RMS acceleration is

$$\sigma_x \equiv [\Delta f_{\text{eff}} \times ASD_{\text{max}}]^{1/2} = [0.5 \pi f_0 Q S_{AA}(f_0)]^{1/2} \quad (7)$$

where  $S_{AA}(f_0)$  is the acceleration spectral density of the input excitation at frequency  $f_0$ . This RMS acceleration response includes the effect of response at all frequencies above and below resonance. The expression in Equation (7) is exactly identical to Equation (5) for a SDOF base excitation system. However, if the actual acceleration spectral density curve in Figure 3 is integrated graphically, it is found that this RMS acceleration corresponds approximately to the area under the curve up to a frequency of approximately  $2f_0$ . Thus, the RMS acceleration as calculated will be in error if there are any additional resonant peaks, i.e., two or multi-degree-of-freedom vibratory systems, at frequencies less than  $2f_0$ . This upper frequency limit will decrease for higher values of  $Q$ .

For a system with force limited excitation, and if the force limit value as defined from Equation (3) is less than the maximum reaction force, the response force spectral density  $FSD_x$  will be reduced or notched to the controlled force limiting value as shown in Figure 4. The depth of the force notching is

$$A^2 = Q^2 / C^2 \quad (8)$$

The notched RMS response,  $\underline{\sigma}$ , of the force-limited excitation will be equal to the total integration of the remaining shaded area. No closed form solution is available to represent the integration result. The approximate values can be calculated by utilizing the results previously described in the random vibration textbook by Crandall and Mark [Ref. 10] and also later reproduced in Hendrickson's approximate formula [Ref. 11]. The results obtained from the Hendrickson's formula are shown in Figure 5, where the fraction of mean square notched response,  $f_r$ , of a SDOF system is plotted against the notching value.

To explain the above prediction process, consider the following example, for a  $Q=10$ , and  $f_0 = 80$  Hz system, and by chosen the force limit factor  $C^2=5$ , the notching depth due to force limiting can be computed and is equal to  $10^2/5 = 20$ , which corresponds to a 13 dB notch. Thus, using Figure 5, the fraction of mean square is approximately equal to  $f_r = 0.28$ . The RMS acceleration response is then equal to 53 percent of the unnotched value. Assume that the weight of the equipment is 20 Kg and is subjected to a  $0.2 \text{ g}^2/\text{Hz}$  input acceleration spectral density. Therefore, the RMS C.G. acceleration response for the force limited excitation would be approximately

$$\underline{\sigma}_{C.G.} \equiv 0.53 \times [0.5 \pi f_o Q S_{AA}(f_o)]^{1/2} = 0.53 \times 15.8 = 8.4 \text{ g}$$

For this example, this corresponds to a 3-sigma peak acceleration of  $3 \times 8.4 = 25.2 \text{ g}$ , which is exactly the acceleration value given by the typical MAC curve as shown in Figure 1.

### Structural Loads Prediction in Force-Limited Vibration Testing

The above example indicates that the mathematical expression of the acceleration response of the force-limited excitation can not be defined a-priori, but the RMS responses have to be evaluated case by case, due to its dependence on several variable factors like: random input excitation levels, equipment resonant frequencies, system damping values, and force limit factors, i.e.,

$$\underline{\sigma}_{C.G.} = F \{ Q, f_o, C, S_{AA} \} \quad (9)$$

For the purpose of investigating the effects of each individual parameter, the complexity of the notched acceleration response requires that the computed results be presented in tabular form for different resonant frequencies of the SDOF verse the structural damping values. Table 1 lists the 3-sigma peak acceleration responses of the force-limited excitation of various SDOF vibratory systems, subjected to the same input acceleration and force limit factor, but with different Q's, and the system resonant frequency,  $f_o$ . As can be seen from the resulting table, the notched acceleration response shows no significant difference for Q variations, i.e., it is practically independent of the damping factor Q of the single resonant mode system. In Figure 6 the results for different force limit factors,  $C^2$ , are plotted against the system's resonant frequency. In comparison with the plotted curve values, the notched RMS value increases more or less proportionally to the square root of the force limit factor, C, in all frequency range. Therefore, an approximate formula to predict the notched RMS response value for a single mode system can be formulated and is expressed as follows:

$$\underline{\sigma}_{C.G.} \equiv [2 C f_o S_{AA}(f_o)]^{1/2} \quad (10)$$

The constant factor  $\sqrt{2}$  is obtained semi-empirically by best matching all computed values for different SDOF vibratory systems. A good agreement between the computed peak acceleration responses and the estimates based on Equation (10) is demonstrated in Table 2.

### Concluding Remarks

- A simplified formula for predicting the equipment structural test loads for force-limited vibration testing has been derived. By properly choosing the force limit factor along with the equipment predominant resonant frequency, the force-limited vibration test will result in test loads comparable with the structural design limit criteria derived from the MAC. This prediction formula is an extremely useful tool that allows the test engineer to estimate equipment structural responses and test induced loads before testing is conducted.

- The structure test loads prediction method presented herein is so far considered to be of a semi-empirical nature, since no rigorous mathematical derivation has been given yet. A parametric investigation of the notched prediction has also confirmed the dependence of the response on typical dynamic parameters like: force limit factors, component resonant frequencies, and random vibration excitation levels, but the response is independent from the test system damping values.
- As noted herein the RMS acceleration computation is correct for a single resonant mode system, but will be in error for two or multi-degree-of-freedom systems. Nevertheless, the results obtained from SDOF systems to random excitations will be the foundation for analyzing response of more complex structural systems subjected to random excitations. In order to complement the development and to assess the validity of the simplified formula for more general usage, the work is currently being extended to study the additional contribution due to other resonant frequency responses in the vicinity of the predominate resonant mode.

## Acknowledgments

The work described in this paper was carried out by the Jet Propulsion Laboratory (JPL), California Institute of Technology under a contract with the National Aeronautics and Space Administration (NASA). The many contributions of the personnel in the JPL Dynamics Environments Group and its supervisor, Dennis Kern, are appreciated. Moreover, the author would like to acknowledge the significant contributions of Dr. Terry Scharton for his development of force limiting technology and many helpful suggestions in preparing this paper and to Ben Thoma for computing the tabular results.

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Table 1. 3-Sigma Peak Acceleration of Single Resonant Mode Systems Subjected to  $0.2 \text{ g}^2/\text{Hz}$  Base Excitation with Force Limit Factor,  $C^2 = 5$

Q \ f <sub>o</sub>	System Resonant Frequency (Hz)							
	20	40	60	80	100	120	150	200
50	12.69	17.94	21.97	25.37	28.37	31.08	34.74	40.12
33	12.68	17.94	21.97	25.37	28.36	31.07	34.74	40.11
25	12.68	17.93	21.96	25.36	28.36	31.06	34.73	40.10
20	12.68	17.93	21.95	25.35	28.34	31.05	34.71	40.08
10	12.64	17.87	21.88	25.27	28.25	30.95	34.60	39.96
5	12.47	17.63	21.60	24.94	27.88	30.54	34.15	39.43
3.3	12.16	17.20	21.07	24.33	27.20	29.79	33.31	38.46

Table 2. Comparison of Computed Peak Accelerations with Estimated Results from Eq. (10) in  $0.2 \text{ g}^2/\text{Hz}$  Force-Limited Base Excitation

C <sup>2</sup> \ f <sub>o</sub>	System Resonant Frequency (Hz)							
	20	40	60	80	100	120	150	200
2	10.09 (10.09)	14.26 (14.27)	14.47 (17.48)	20.17 (20.18)	22.55 (22.56)	24.71 (24.72)	27.62 (27.63)	31.90 (31.91)
3	11.16 (11.17)	(15.78) (15.79)	19.33 (19.34)	22.32 (22.33)	24.91 (24.97)	27.34 (27.35)	30.56 (30.58)	35.29 (35.31)
4	11.99 (12.00)	16.96 (16.97)	20.77 (20.78)	23.98 (24.00)	26.81 (26.83)	29.37 (29.39)	32.84 (32.86)	37.92 (37.95)
5	12.68 (12.69)	17.93 (17.94)	21.95 (21.98)	25.35 (25.38)	28.34 (28.37)	31.05 (31.08)	34.71 (34.75)	40.08 (40.12)

( ) Estimated results from Eq. (10)



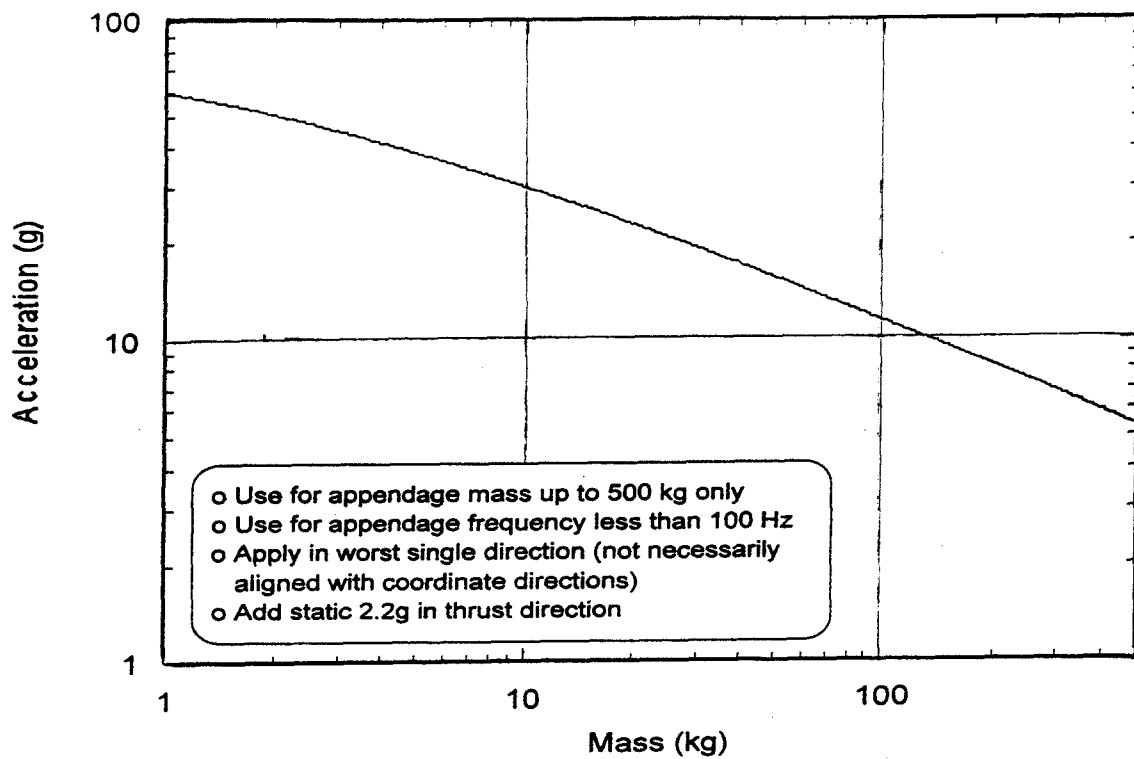


Figure 1. Typical Physical Mass Acceleration Curve for Flight Equipment

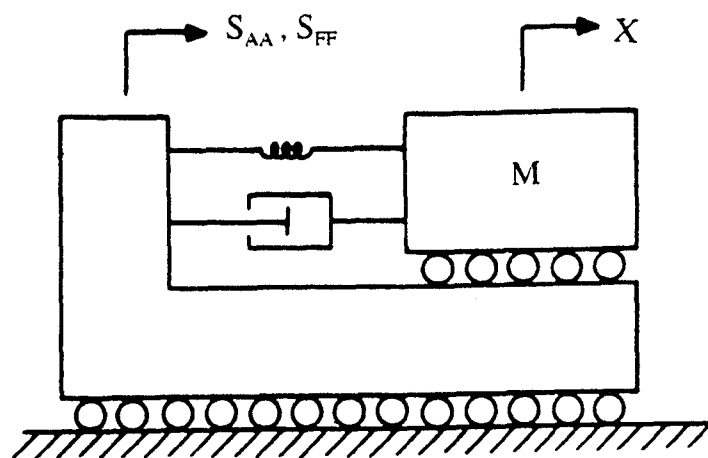


Figure 2. Single-Degree-of-Freedom System Driven by Base Random Excitation

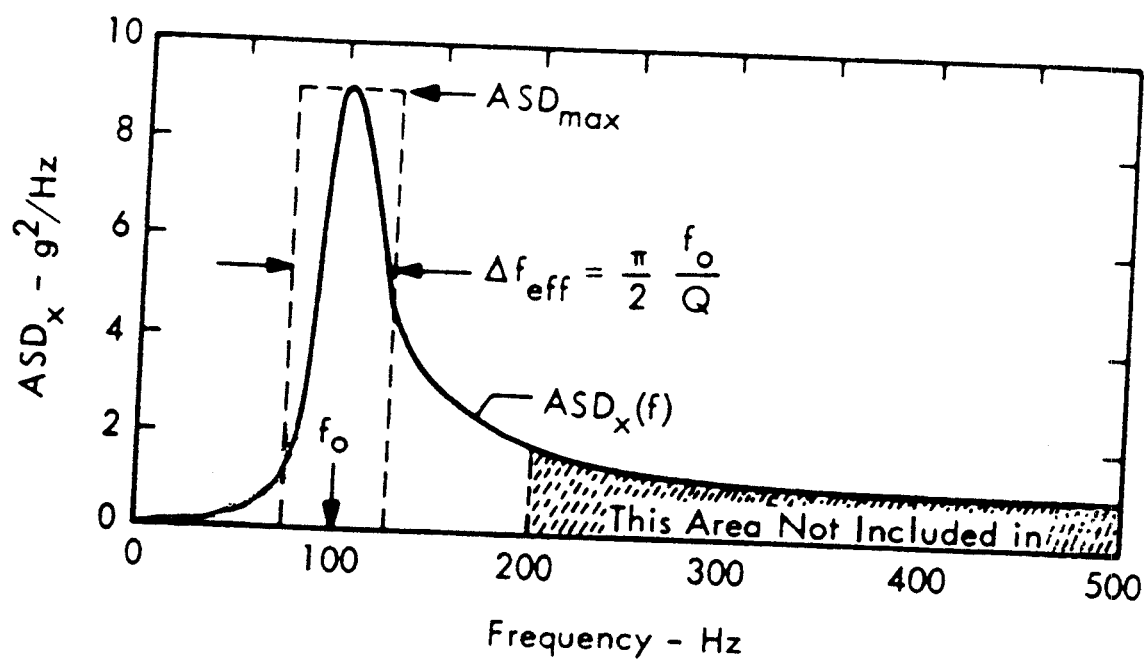


Figure 3. Acceleration Spectral Density Curve for Response of Single Resonant Mode Systems

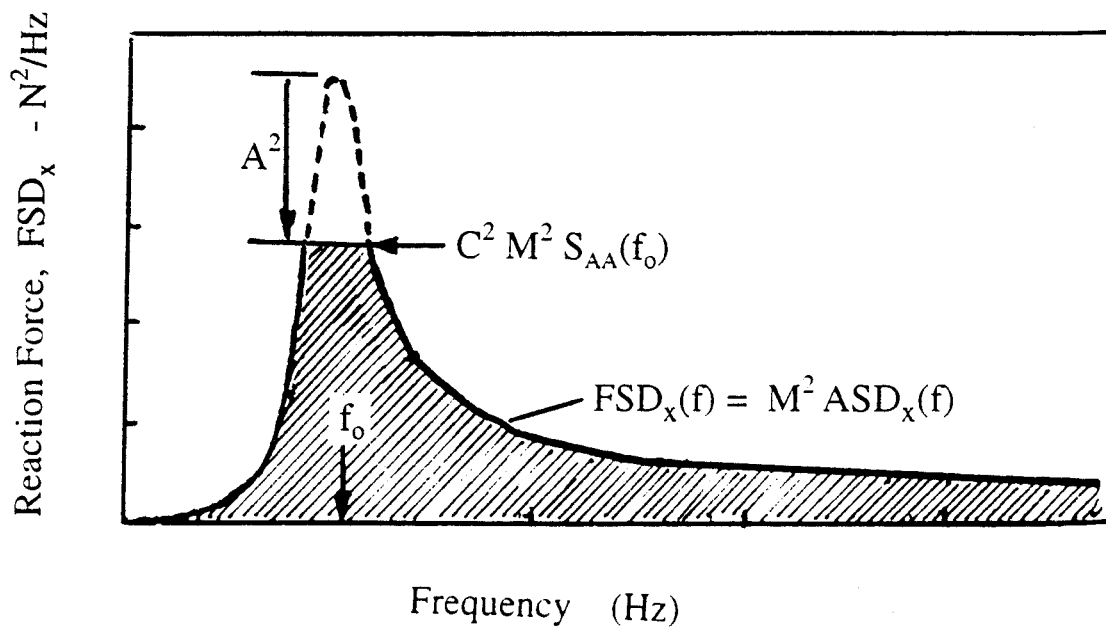


Figure 4. Force-Limited Responses of Single Resonant Mode Systems

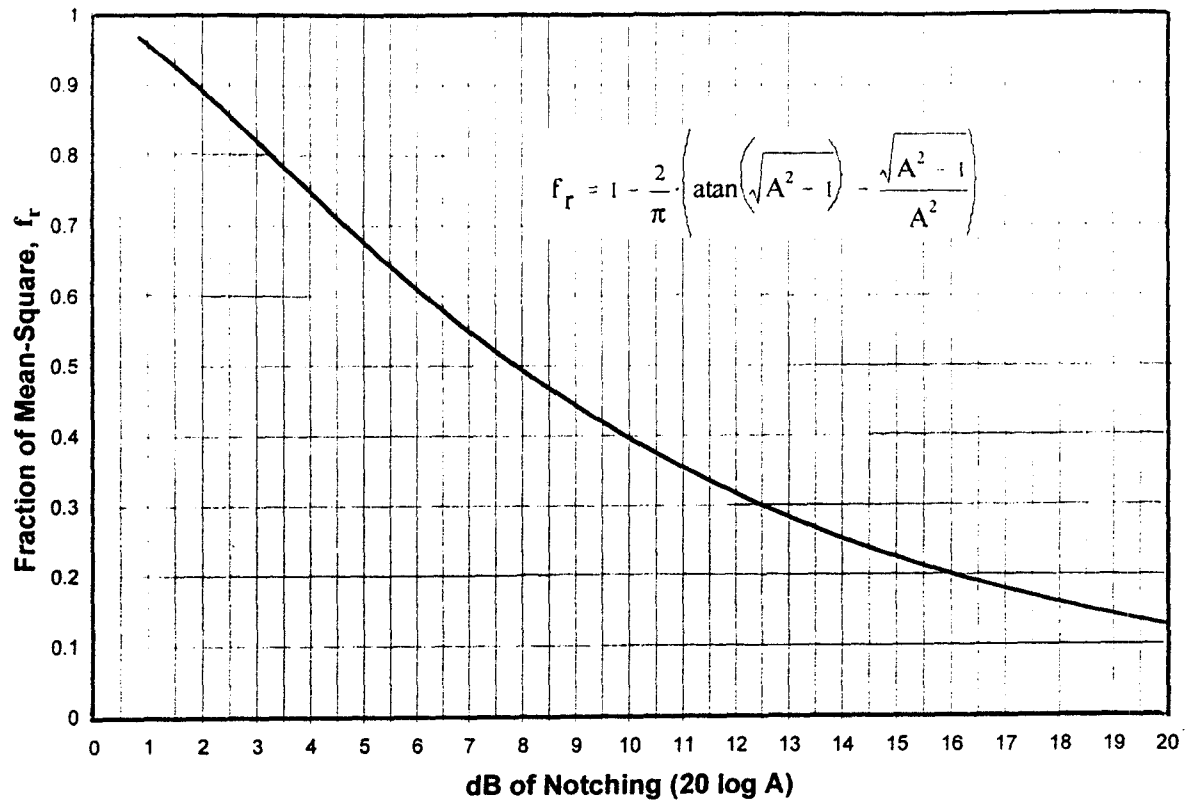


Figure 5. Notched Mean-Square Responses of SDOF Systems Subjected to Base Random Excitation

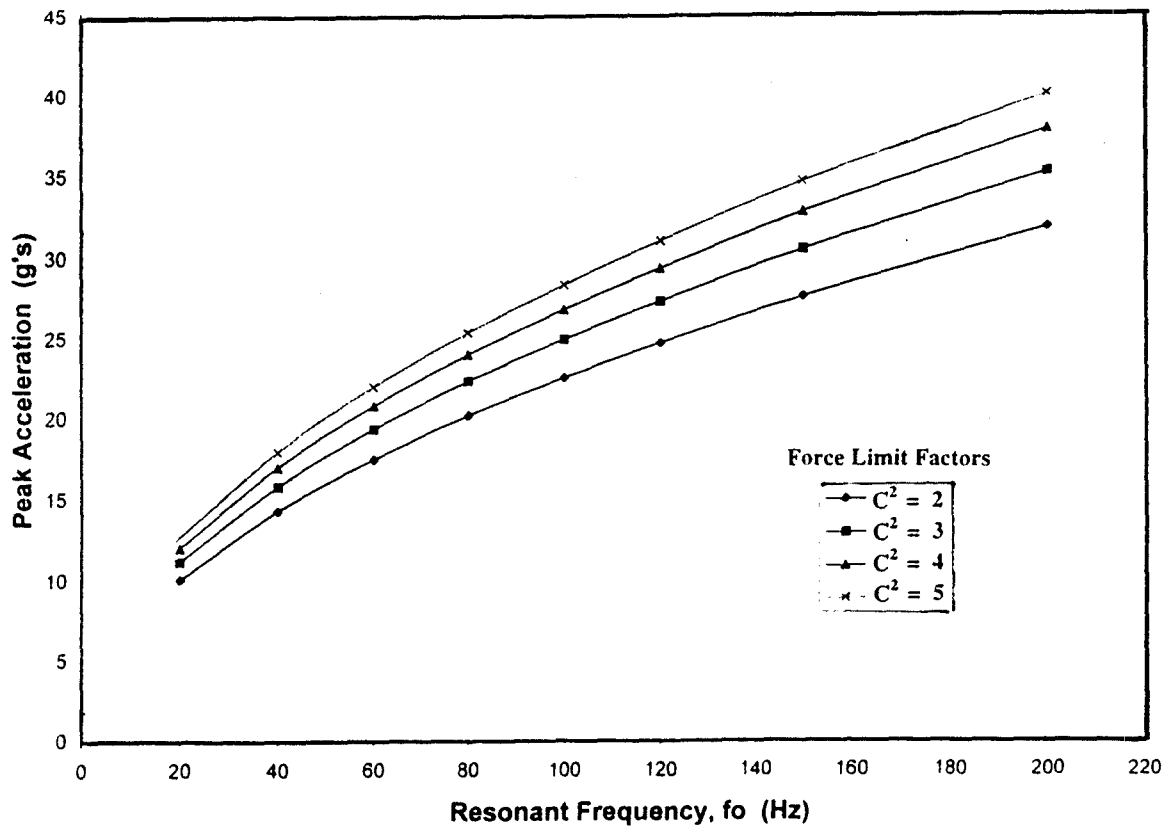


Figure 6. 3-Sigma Peak Accelerations of Single Resonant Mode Systems Subjected to  $0.2 \text{ g}^2/\text{Hz}$  Force-Limited Base Excitation